# Dual Dig Level I (2008)

1. Solve for x and y, where x and y are real numbers, and $i = \sqrt{-1}$: $(2x + 5y) + (3x - 2y)i = 19i$.

2. Simplify: $\sqrt{\dfrac{4}{3}} - \sqrt{\dfrac{3}{4}}$

3. Simplify: $\dfrac{\log 15}{\log\left(\dfrac{1}{15}\right)}$

4. How many integers between 1 and 1000 are divisible by 3 but are not divisible by 4?

5. Solve for a in terms of b and c if $\dfrac{1}{a} = \dfrac{1}{b} + \dfrac{1}{c}$. Assume that a, b, and c are nonzero.

6. Let $f(x)$ be a periodic function with a fundamental period of 5. If $f(1) = 3$, $f(2) = 4$, and $f(3) = 5$, find the exact value of $f(72)$.

7. Find the equation of a quadratic function if its graph in the standard xy-plane has x-intercepts of 5 and -3 and a y-intercept of -8.

8. A chemist is mixing solutions of acid together. How much pure water should be mixed with a 40% solution of acid to produce 20 liters of a 10% acid solution?

9. The triangle shown is a **right** triangle and each side of the triangle is the diameter of a semicircle. For simplicity, we will refer to each semicircle by its diameter. If semicircle Z has an area of 169 m^2 and semicircle X has an area of 25 m^2, then what is the exact length of segment Y?
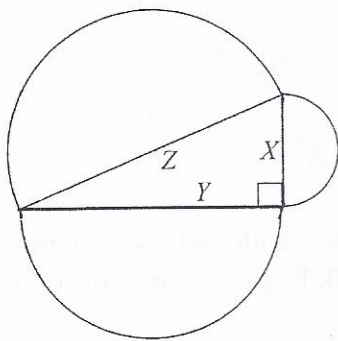
10. A farmer raises only chickens and cows. There are 34 animals in all. One day, after falling on the ground, the farmer counts 110 legs total on all of his animals. How many chickens are there?

11. Simplify $(1 + i)^{16}$ completely, where $i = \sqrt{-1}$.

12. Solve: $x + 6 \le 3x - 4 \le x^2 - 8$. Write your answer in interval form.

13. A rubber ball is blasted straight up into the air. Disregarding air friction, the spin of the earth, etc., the height of the ball is given by: $h(t) = -16t^2 + vt + c$, where $h(t)$ represents the height of the ball (in feet from the ground) at time t (in minutes after the blast), and v represents the initial velocity, while c represents the initial height from which the ball is thrown. If the initial velocity is 64 feet/minute, and the ball is thrown from the top of a 176 ft tall building, at what time(s) [after the blast] is the height of the ball 96 feet?

14. Let $f(x) = \dfrac{cx}{2x+3}$, where $x \neq -\dfrac{3}{2}$. Find all real values of c, if any, for which $f(f(x)) = x$.

15. Al and Betty each pick a positive integer. The least common multiple of their integers is $2^3 \cdot 3^4 \cdot 5 \cdot 7$. The greatest common divisor of their integers is $2 \cdot 3 \cdot 5$. We find out that Al picked 210. What is Betty's integer?

16. Find all real solutions of: $6^{1+x} + 6^{1-x} = 37$.

17. In how many ways can $10 be changed into just dimes and quarters, provided that at least one of each coin must be used?

18. The following five numbers are placed into a jar and shaken: 71, 76, 80, 82, and 91. The numbers are drawn out, one by one, in such a way that the average of the first two numbers drawn is an integer, the average of the first three numbers drawn is an integer, and the average of the first four numbers drawn is an integer. What must be the third, fourth, and fifth numbers drawn? Hint: Consider the "three numbers drawn" case first by using remainders when the numbers are divided by 3.

19. In the figure below, equilateral triangle DEF is inscribed in equilateral triangle ABC. We have that $\overline{EF} \perp \overline{BC}$. What is the ratio of the area of triangle DEF to the area of triangle ABC? Write your answer in simplest form.
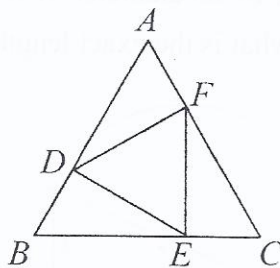
20. Consider the polynomial $x^3 + ax^2 + bx + c$ with real coefficients, where all three roots (or zeros) are real, and two of the roots have sum equal to 0. Express c in terms of a and b.